Proposed comps topic(s) 2020-21 (Mark Krusemeyer)

Each of the projects below is an opportunity for two people to delve deeply into material beyond the standard undergraduate curriculum, and to get substantial (weekly) experience in presenting that material at the blackboard. (Quite likely, only one of the projects will be run.) It is conceivable that a topic will be modified, or even changed altogether, based on participants' interests or background.

Directed reading in elliptic functions and modular forms

We will start by looking at elliptic functions, which are functions of a complex variable that have two independent periods - unlike trigonometric functions, which have only one. This will lead us to Eisenstein series, modular forms, the "classical" proof of the formula

$$\sigma_7(n) = \sigma_3(n) \,+\, 120 \sum_{k=1}^{n-1} \sigma_3(k)\sigma_3(n-k),$$

where $\sigma_r(n)$ denotes the sum of the r-th powers of the divisors of n, and more beautiful mathematics than we can possibly get through.
Sources will likely include one or more of Apostol, *Modular Functions and Dirichlet Series in Number Theory*; Koblitz, *Introduction to Elliptic Curves and Modular Forms*; Serre, *A Course in Arithmetic.*
Terms: W, S.
Prerequisites: (Required) Math 361 or 261; (highly recommended) Math 342. Some background in number theory may be helpful, but is not required.

Directed reading in algebraic geometry

In algebraic geometry, techniques from abstract algebra are used to describe and investigate "varieties": sets that are defined by polynomial equations (in several variables). The interplay between algebraic and geometric points of view makes this a very rich topic, that has expanded enormously in the last hundred years or so; while it is traditionally considered "pure" mathematics, the more computational parts have applications in such areas as robotics and computer-aided design. The first major goal of the reading would almost certainly be Hilbert's "Nullstellensatz" (theorem of the zeros), which establishes a correspondence between varieties and certain ideals in polynomial rings. After that there is a variety (pun intended) of possibilities, depending on the background, interests, and tolerance for abstraction of the participants. Those factors will also help determine the choice of sources for this project.
Terms: W, S.
Prerequisite: Math 342. Additional background in algebra may be helpful, but Math 352 is not required; in particular, we are not likely to use anything substantial from either Galois theory or representation theory (the most frequent topics in Math 352 in recent years).